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LINEAR AND NON-LINEAR, LOCAL AND GLOBAL STABILITY ANALYSIS OF OPEN FLOWS

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1. Preamble

The present article issues from a course and it is meant to present the physical implications of recent advances in open flow stability theory. The literature will be reviewed briefly to enlighten important points, and the reader is referred to the review by Huerre & Monkewitz 1990 (HM90) for an exhaustive exploration of published results. The present article also departs from the HM90 review in that it includes a discussion of the non-linear dynamics of open flows.

2. Introduction

Following the works by Brown & Roshko (1974) and Winant & Browand (1974) on mixing layers, the description of coherent structure dynamics became a subfield in itself within the field of turbulence. In these experiments, visualizations demonstrated that recognizable structures (vortex billows) keep governing the dynamics of the flow even at high Reynolds numbers. Therefore the quest for a model able to describe the birth, the growth, the decay and the death of such structures became legitimate. Schematically two points of view have been developed, one representing each coherent structure by an ad hoc model in real space (vortex billows, hairpin vortices, plumes), the other considering a series of coherent structures as the non-linear evolution of an instability wave. Mixing layers, wakes, boundary layers, belong to the open flow category where particles of fluid enter and leave the experimental domain continuously. Therefore the desirable theory of instability waves should take into account the open flows peculiarities : the spatial origin of the flow, the inflow perturbations (extrinsic noise), the mean advection and the spatial evolution of the mean flow (spatial inhomogeneity). It should describe the intrinsic turbulence as well as the extrinsic (noise driven) turbulence and should allow us to discriminate between these two classes of turbulent flows.

This theory should also be fully non linear as experimental flows present strong instability (high Reynolds number experiments). The present state of the art is far from reaching such an ambitious goal. Bifurcation theory, which consists in following the series of transitions affecting a flow when the Reynolds number is increased from zero, may provide a way to achieve this goal. On the apparently simple problem of the cylinder wake, this theory already fails to characterize the first non stationary bifurcation describing the occurrence of the well-known Karman vortex street. The difficulties come from the fact that the operator describing the stability of the stationary basic state is not self-adjoint (Collet 1992) on account of the presence of the advection term $U\partial/\partial x$ (U the velocity difference between the cylinder and the far fluid, x the direction of U). We will show in the following that, for the same reason, a non linear extrapolation of linear global stability results turns to be out of the reach of classical techniques.

3. Mathematical Framework

The present paper address mainly one dimensional (1D) problems with a single space variable x. The way such a 1D system is related to a fluid mechanics problem is described convincingly in HM90. Fluctuations around a basic flow are split into elementary instability waves $A \exp i(kx - \omega t)\varphi(y, k, \omega)$ of complex wave number k and complex frequency ω. The existence of $\varphi(y, k, \omega)$ constrains k and ω to satisfy a dispersion relation of the form :

$$D(k, \omega, R) = 0 \ . \tag{1}$$

Therefore the original problem is equivalent to solving the integro-differential equation

$$D(-i\frac{\partial}{\partial x}, i\frac{\partial}{\partial t}, R)\phi(x, t) = F(x, t) \ , \tag{2}$$

where $F(x, t)$ represents the forcing term. The forcing is equal to zero for plane waves.

A practical way to introduce the non-linearity is to let D depend on ϕ. This is justified in the amplitude equation framework (Manneville 1991) which is valid close to critical. As often in the amplitude equation field, we will study the equation out of its domain of validity in a qualitative manner. The solution of the simplified 1D problem is then thought to be representative of a wider class of non-linear problems which are not tractable otherwise. The only difference with amplitude equation theory is that we keep the carrier wave $\exp i(kx - \omega t)$ as we want to treat the inhomogeneous problem.

The spatial development of the basic flow is introduced in the same way as in HM90 with an evolution length scale L defined as

$$\frac{1}{L} = \theta \frac{d\theta}{dx} , \tag{3}$$

where θ is, say, the local momentum or vorticity thickness. The main parameter $\epsilon = \frac{1/L}{k}$ measures the degree of inhomogeneity of the flow. For $\epsilon \ll 1$ the basic flow changes over a slow space scale $X = \epsilon x$ and, by a WKBJ type of analysis, it is possible to show that the fluctuations $\phi(x,t)$ now satisfy :

$$\mathcal{D}(-i\frac{\partial}{\partial x}, i\frac{\partial}{\partial t}, \theta(X), \epsilon\frac{d\theta}{dX}(X), R)\phi(x,t) = F(x,t) . \tag{4}$$

As for homogeneous flows, non-linearities may be introduced by making the operator $\mathcal{D}$ depend on ϕ. Extension of this equation to large amplitude, to strong non-linearities and to important non-parallelism (finite ϵ), allows us to qualitatively explore the possible behaviour of an inhomogeneous non-linear system. Its pertinence to real fluid dynamics experiments may only be conjectured and not rigourously traced back. The reader is referred to Monkewitz *et al* (1992) and Le Dizès (1992) for a complete account.

4. Linear Theory For Homogeneous Flows

Instability waves are governed by equation (1). Temporal modes $\omega(k, R)$ refer to cases where the complex frequency ω is determined as a function of real wave number k. Conversely, spatial branches $k(\omega, R)$ are obtained by solving (1) for complex wave numbers k when ω is given real. Spatial instability theory is important when the response to a localized forcing is of interest. In particular this is the case in the problem of noise forcing at the spatial origin of an open flow.

The difference between time t and space coordinate x comes from the causality assumption and is discussed in Chomaz *et al* 1991 (CHR91). Starting the "experiment" at a given instant, say $t = 0$, breaks the symmetry $t \rightarrow -t$ and the causality condition stipulates that no response should be observed before $t = 0$. Thus the dispersion relation D should have the property sketched in figure 1 : schematically the temporal branches lie under a contour deformed from the real axis $\omega_i = 0$ whereas spatial branches are not constrained.

For a stable system, temporal modes lie under the real ω axis (Fig. 1a1). No ω real and k real is solution of (1). Therefore spatial branches do not cross the real k axis (Fig. 1b1) and lie on each side of the real k axis. As the flow is stable we only allow a damped response to forcing. Thus branches in the upper k domain must be placed on the right of the source carrying energy to $+\infty$. Similarly branches in the lower half k plane shall be associated to left going waves (Fig. 1c1).

When the basic state is destabilized one temporal branch crosses over the real ω axis (Fig. 1a2). Intersection of this temporal branch with $\omega_i = 0$ gives two waves pertaining to a spatial branch as well. Therefore destabilization is associated with

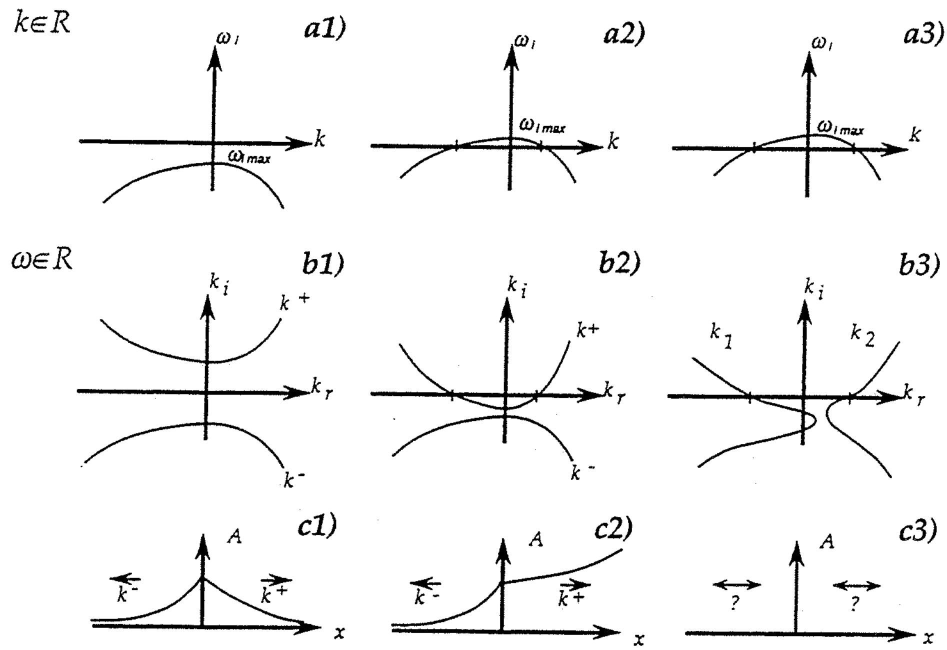

Figure 1. a) Temporal modes $\omega(k)$, k real ; b) spatial branches $k(\omega)$, ω real, in k-plane ; c) response to forcing localized in space and harmonic in time. (1) stable case ; (2) convectively unstable case ; (3) absolutely unstable case.

the crossing of the real k axis by one spatial branch (Fig. 1b2). By continuation argument, as no topological change occurs on the spatial branches at this transition, we are allowed to keep the same energy propagation direction to each branch as in the previous case (Fig. 1c2).

This continuation argument stops to be true when two spatial branches issuing from the upper and lower k plane collide (Fig. 1b3). In this case, no reasonable argument allows us to define the response to the source as no energy propagation direction may be attributed to the various spatial branches (Fig. 1c3). It should be noticed that no pathological behaviour is observed on the temporal branches (Fig. 1a3). This transition happen when the saddle point, $\frac{d\omega}{dk} = 0$, of the dispersion relation, which is located at k_0 and ω_0, is such that $\omega_{0i} = 0$.

In conclusion, if $\omega_{i\ max}$, the maximum imaginary part of the temporal mode, defines through its sign the stability of the flow, an other quantity, ω_{0i}, named the absolute growth rate, determines the ability to define the response to a localized source. When ω_{0i} is negative, this response is defined and the flow is said to be convectively unstable. If ω_{0i} is positive, the instability is absolute and the response to forcing cannot be defined.

This derivation "by hand" is in fact the physical counter-part of classical absolute/convective instability theory (see HM90 for references) which makes use of causality and contour deformation argument to compute the signaling problem.

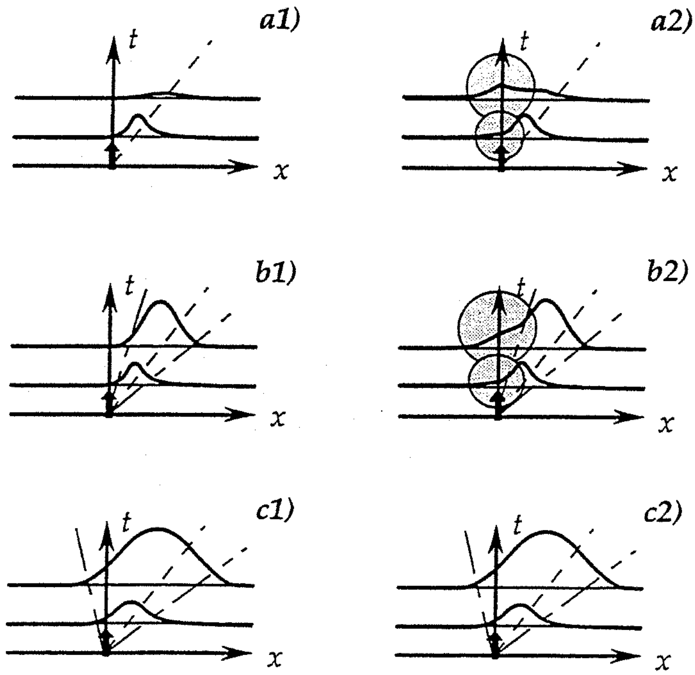

Figure 2. (x,t) diagrams showing : 1) The impulse response and 2) The response to a causal forcing which starts at $t = 0$ in homogeneous media : (a) stable ; (b) convectively unstable ; (c) absolutely unstable. Only in the stable and convectively unstable case the response to a steady forcing may be determined (shaded region).

This subtle effect of causality makes even more sense when looking at the differences in the (x, t) diagrams representing the impulse response in the absolute and convective cases, together with the (x, t) diagrams representing the response to a forcing impulsively started at time $t = 0$ (Fig. 2). The quantity $\omega_{i\ max}$ is observed to describe the growth of the wave packet maximum. Whereas ω_{0i} describes the growth of the wave packet at a fixed location. Only in the absolute instability cases (Fig. 2c), the wave packet grows exponentially at a fixed location and over-shadow the response to the forcing. The above considerations demonstrate that convectively unstable open flows behave as spatial amplifiers of incoming perturbation. On the contrary absolutely unstable flows follow intrinsic dynamics.

5. Global Linear Stability Analysis of Weakly Non-Homogeneous Flows

As we already stated in the introduction, non uniformity in x is one of the features of open flows. In this case only the translation in time is left as an invariance of the basic state. Stability analysis has to be performed globally on the whole domain by looking for solutions of the form $\exp(-i\omega_G t)\phi_G(x)$. The term "global" does not refer here to finite amplitude effects in phase space but to the fact that x, now, is to be considered as an eigenfunction direction. The purpose of the present section is to emphasize important results that link the local instability characteristics at each streamwise x-station and the global instability properties over many wavelengths of the instability. A pioneering work on this subject is due to Soward & Jones (1983) who, however, do not comment on the link between $\omega_0(x)$ and ω_G i-e between the existence of a pocket of absolute instability and the occurrence of an amplified global mode. Analyses of model problems have been made in CHR (1987, 1988) and general theorems have been derived in CHR (1990) and extended in Monkewitz *et al* (1992) and Le Dizès (1992). In my view the most important and robust theorem is that a pocket of absolute instability, somewhere in the flow, is necessary in order to sustain a global instability. The fact that we only have a necessary condition has been wonderfully illustrated by Hunt & Crighton (1991) who exhibit a model problem where the flow is everywhere absolutely unstable but globally stable. The second result gives a quantitative prediction of the global mode frequency under certain regularity assumptions for the dispersion relation. At leading order, ω_G is given by $\omega_0(X_S)$ such that $\frac{\partial\omega_0}{\partial X}(X_S) = 0$ or equivalently $\omega(k_S, X_S, R)$ with

$$\begin{cases} \dfrac{\partial\omega}{\partial k}(k_S, X_S, R) = 0 \ , \\ \dfrac{\partial\omega}{\partial X}(k_S, X_S, R) = 0 \ . \end{cases}$$

In this case the structure of the global mode is similar to the one computed by Soward & Jones (1983). I must say that we first overlooked and then underestimated Soward & Jones' contribution because it pertains to a closed flow geometry. It still enters the absolute and convective framework for the subtle reason that the instability breaks the $x \to -x$ symmetry of the basic state. A similar phenomenon occurs in convecting binary mixtures (Cross (1986-1988), Heinrich *et al* (1987), Roses *et al* (1987), Fineberg *et al* (1988), Kolodner & Surko (1988)).

6. Weakly Non-Linear Global Stability Analysis of Non-Homogeneous flows.

Weakly non-linear global stability analysis has been discussed in CHR90 for model amplitude equations. A more complete study including extension to real fluid dynamical problems will appear in Le Dizès (1992). The problem turns out to be difficult and even ill-posed when the WKBJ approximation is made. The trouble originates from the presence of the advection term $U\frac{\partial}{\partial x}$ in the integro-differential operator L which makes L non self-adjoint. A deep physical reason is hidden behind this technical issue. The adjoint operator L^A allows to trace back the cause when the effect is given . The solution ψ_G that belongs to the Kernel of L^A defines the receptivity of the flow. As the advection acts in the opposite direction for the adjoint problem, ψ_G admits a maximum well upstream of the most unstable region. For the same reason the eigenfunction ϕ_G of the original problem (the global mode) has a maximum shifted downstream.

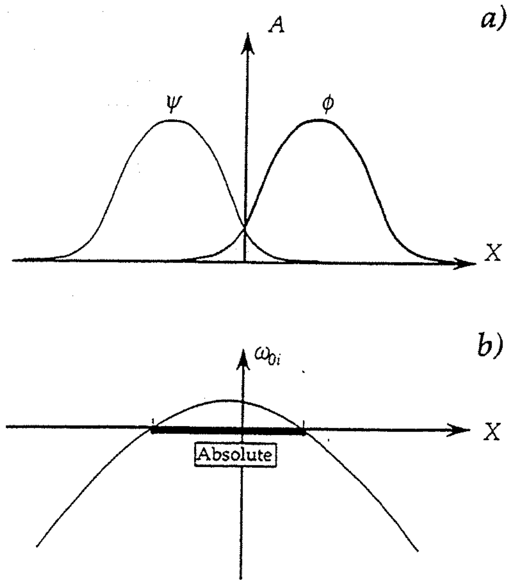

Figure 3. a) Sketches of a global mode envelope ϕ_G in space and of the solution of the adjoint problem ψ_G. ϕ_G may be interpreted as the shape of the self excited mode and ψ_G as the measure of the magnitude of the global response as a function of the location of the forcing source. b) the corresponding ω_{0i} variations. Comparing ϕ_G, ψ_G and ω_{0i} one observes that the absolutely unstable region is not associated with special properties on the forcing efficiency or on the response magnitude.

In conclusion the flow is sensitive in a region where the amplitude of the solution is small (even exponentially small) in the WKB approximation (for order 1 advection velocity). Non-linear saturation comes usually from reinjection of energy to the basic state by local non-linearity which here occurs close to the maximum of ϕ_G, in the downstream region where the flow is insensitive to forcing (Fig. 3)!

This heuristic argument is confirmed by the exact results obtained on the subclass of systems defined by :

$$\frac{\partial A}{\partial t} + L\left(\frac{\partial}{\partial x}, x, R\right) A + c(x,R) \mid A \mid^2 A = f(x,t) \ , \tag{5}$$

with A, $c(x,R)$ complex. The detailed discussion and derivation may be found in CHR90. Only a summary is given in the following. We suppose that $A(x,t)$ vanishes at the boundaries of the finite or semi-infinite domain. The linear homogeneous equations, associated with (5), admits a solution of the form :

$$A_G(x,t) = \phi_G(x) \exp(-i\omega_G t) \ ,$$

for the gravest mode. The frequency ω_G and modal function $\phi_G(x)$ depend on R and there exists a critical value R_c such that the system is globally stable for $R < R_c$. At $R = R_c$ the system is neutral with $\omega_{Gi}(R_c) = 0$. In this case, the amplitude of the linearized marginal mode is unconstrained and may evolve slowly with respect to the time scale ω_G^{-1}, which we suppose is finite.

A multiple-scale analysis is performed using the small parameter η which measures the departure from criticality :

$$\begin{cases} R = R_G + \eta^2 \Delta_R \ , \\ f(x,t) = \eta^2 F \delta(x - x_f) \exp(-i\omega_f t) \ , \\ \omega_f = \omega_G + \eta^2 \Omega \ . \end{cases} .$$

A slow time scale $T = \eta^2 t$ is introduced in order to avoid the appearance of secular terms in the perturbation expansion

$$A(x,t) = \sum_{n=1}^{\infty} \eta^n A_n(x,t,T) \ .$$

Assuming that the operator L is analytic in the parameter R we may write

$$L\left(\frac{\partial}{\partial x}, x, R\right) = L\left(\frac{\partial}{\partial x}, x, R_G\right) + \eta^2 \Delta_R L_R \left(\frac{\partial}{\partial x}, x, R_G\right) + O(\eta^4) \ .$$

Thus, the leading order term is described by the equation

$$\frac{\partial A_1}{\partial t} + L\left(\frac{\partial}{\partial x}, x, R_G\right) A_1 = \frac{\partial A_1}{\partial t} + L_G A_1 = 0 \ , \tag{6}$$

with the solution

$$A_G(x,t) = \mathcal{A}(T) \phi_G(x) \exp(-i\omega_G t) \ ,$$

defining the neutral global mode with arbitrary amplitude $\mathcal{A}(T)$. The next order term A_2 satisfies the same homogeneous equation as (6) and provides no essential information or constraint concerning $\mathcal{A}(T)$. A compatibility condition for the avoidance of secular therms at third order leads to the following evolution equation for the global mode amplitude :

$$\begin{aligned}\frac{d\mathcal{A}}{dT} =&\Delta_R \frac{< \psi_G | L_R \phi_G >}{< \psi_G | \phi_G >} \mathcal{A} \\ &- \frac{< \psi_G | c(x,R) | \phi_G |^2 \phi_G >}{< \psi_G | \phi_G >} |\mathcal{A}|^2 \mathcal{A} - F \frac{\psi_G^*(x)}{< \psi_G | \phi_G >} \exp(-i\Omega_G T) \ .\end{aligned} \tag{7}$$

The quantity $< f | g >$ denotes the scalar product $\int f^* g dx$, superscript* denotes the complex conjugate, and $\psi_G(x)$ is the solution of the equation

$$\left(i\omega_G^* + L^A \left(\frac{\partial}{\partial x}, x, R_G \right) \right) \psi_G = 0 \ ,$$

where L^A is the adjoint of L. Figure 3 represents the eigenfunction of the operator and its adjoint for the explicit case defined in CHR90. From the last term of equation (7) we see that ψ_G indeed defines the forcing efficiency which is greater upstream of the most absolutely unstable region. The best way to control the flow is therefore to introduce forcing at the maximum of the function ψ_G. The various scalar products, that appears in (7), depend on the overlapping between ψ_G and ϕ_G which tends to zero in the WKBJ approximation. In the model problem with strong inhomogeneities, the non-linear constant of equation (7) is found to be of either sign compared to the sign of the original non linearity. The exact result on the model problem gives a Landau constant which flip sign as the WKBJ parameter ϵ goes to zero (Le Dizès 1992). This demonstrates that the weakly non-linear global stability is ill posed in the WKBJ frame work. A possible reason for the failure of this classical technique, lies in the fact that the flow in the absolutely unstable region is far from neutral and resonances may trigger large response and a large deformations of the global mode even close to the onset of global instability.

Experimental results by Kyle & Sreenivasan (preprint) and numerical results from CHR90 show that the bifurcation to a global mode does not always follow usual scaling laws (amplitude proportional to the square root of the departure from criticality). More experimental, numerical and theoretical results are highly needed on the crucial issues of the non-linear extension of linear global analysis.

7. Non-Linear Stability Analysis of Open Flows

Up to this point only flows which are linearly unstable have been discussed but open flows, such as Poiseuille flow or boundary layer flow, are believed to be non-linearly unstable. Peculiarities of open flows already mentioned should be incorporated in the corresponding non-linear stability analysis. Some numerical studies, meant to illustrate the specific behaviour of non-linear open flows, have been performed by Deissler (see Deissler 1989 for a review). Recently a straightforward extension of the absolute/convective instability concept to non-linear open flows has been proposed (Chomaz 1992).

It appears natural to propose the following definitions of non-linear absolute and convective instabilities : The basic state of a system is stable (S) if, for all initial perturbations of finite extent and finite amplitude, the flow relaxes to the basic state everywhere in any moving frame. A system is unstable if it is not stable in the above sense. The instability is non-linearly convective (NLC) if, for all initial perturbations of finite extent and finite amplitude, the flow relaxes to the basic state everywhere in the laboratory frame. It is non-linearly absolute (NLA) if there exists an initial condition of finite extent and amplitude and a location where the system does not relax to the basic state.

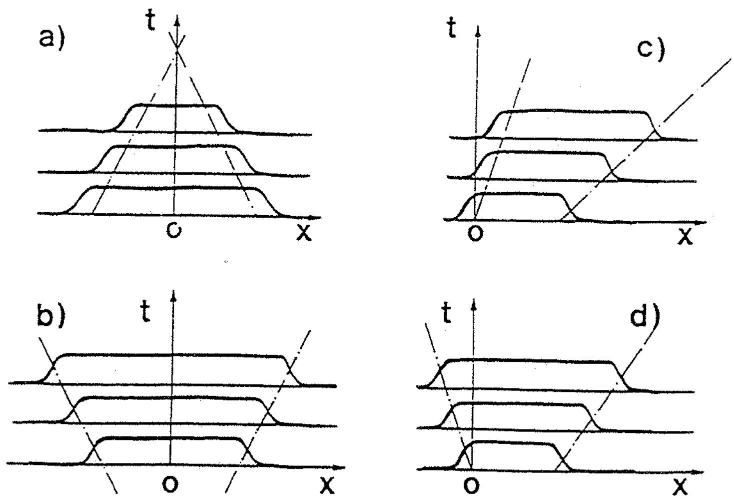

Figure 4. Diagrams in the (x,t) plane, displaying the dynamics of droplets of bifurcated state, (a) A_0 stable, A_2 metastable, (b) A_0 metastable and A_2 stable, (c) non-linear convective instability, (d) non-linear absolute instability.

The physical implication of these concepts has been analysed on the simplest equation exhibiting a subcritical bifurcation :

$$\begin{cases} \dfrac{\partial A}{\partial t} + U\dfrac{\partial A}{\partial x} = \dfrac{\partial V(A)}{\partial A} + \dfrac{\partial^2 A}{\partial x^2}\,, \\ V(A) = R\dfrac{A^2}{2} + \dfrac{A^4}{4} - \dfrac{A^6}{6}\,, \end{cases} \tag{8}$$

where A is a real amplitude and R the control parameter. Experimentally generated open flows may be modelled by an amplitude equation which is to be solved in a semi-infinite domain $[0, +\infty[$ with a suitable boundary condition at $x = 0$ (broken Galilean invariance). The classical theory (see for example Lifshitz & Pitaevskii (1981) or Balian *et al* (1980)) gives the following results for infinite domains. Stable homogeneous solutions correspond to minima of $V(A)$. When $R < -1/4$, $A_0 = 0$ is the only minimum. When $-1/4 < R < 0$, there exist two minima at $A_0 = 0$ and $A_2 = \sqrt{1/2 + \sqrt{R + 1/4}}$. When $R > 0$, $A_0 = 0$ becomes a maximum of $V(A)$, i-e unstable and A_2 is the only stable solution. The parameter value $R_M = -3/16$ defines the Maxwell point at which the solutions A_0 and A_2 have an equal potential density $V(A)$. The relative position of R with respect to R_M determines whether a sufficiently large "droplet" of bifurcated state A_2 surrounded by the basic state A_0 shrinks ($R < R_M$, Fig. 4a) or expands ($R_M < R < 0$, Fig. 4b). The latter case, when considering the advection term, splits into two subcases : an NLC range (Fig. 4c) and an NLA range (Fig. 4d) depending on the respective signs of the two front velocities in the laboratory frame.

The predictions of the potential model (8) in a semi infinite domain $[0, +\infty[$ can be summarized as follows : When a flow is NLC, the only observable steady solution in the absence of forcing, localized at the origin, is the rest state $A = 0$. As the forcing amplitude is increased to A_2 and then decreased back to zero, one observes a hysteresis loop composed of the spatially decaying state asymptotic to $A = 0$ and a spatially growing state asymptotic to A_2. There is, however a reversible return to the rest state $A = 0$ when forcing is turned off. In contrast, when a flow is NLA, both the rest state $A = 0$ and the spatially-growing state asymptotic to A_2 are observable in the absence of forcing. Thus, starting from the rest state, similar variations of the forcing magnitude trigger an irreversible transition to a spatially growing state asymptotic to A_2. The rest state $A = 0$ is not recovered when the forcing is suppressed.

This new angle of view on the stability of non linear open flows deserve a more complete exploration by experimental, numerical as well as theoretical studies.

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References

[1] Balian, R., Klemann, M. & Poirier, J.P., *Physics of defects* North Holland, Amsterdam 1980.

[2] Bers, A., Linear waves and instabilities in *Physique des Plasmas* C. De Witt & J. Peyraud, New York : Gordon & Breach, 117-215 1975.

[3] Briggs, R.J., *Electron-Stream Interaction With Plasmas* Cambridge, Mass. : MIT Press 1964.

[4] Brown, G.L. & Roshko, A., On density effects and large structure in turbulent mixing layers, *J. Fluid Mech* **64** 775-816 1974.

[5] Chomaz, J.M., Absolute and convective instabiliies in non-linear systems, ac-

cepted in *Phys. Rev. Lett.* 1992.

[6] Chomaz, J.M., Huerre, P. & Redekopp, L.G., Models of hydrodynamic resonances in separated shear flows, *Proc. Symp. Turbul. Shear flows, 6th,* Toulouse, France 3.2.1-6 1987.

[7] Chomaz, J.M., Huerre, P. & Redekopp, L.G., Bifurcations to local and global modes in spatially-developing flows, *Phys. Rev. Lett.* **60** 25-28 1988.

[8] Chomaz, J.M., Huerre, P. & Redekopp, L. G., Effect of non-linearity and forcing on global modes, in *Proceedings of the Conference on New Trends in Nonlinear Dynamical and Pattern-forming Phenomena* Coullet, P. & Huerre, P., Eds., NATO ASI Series B : Physics, 237 1990.

[9] Chomaz, J.M., Huerre, P., & Redekopp, L.G., A frequency selection criterion in spatially-developing flows, *Stud. Appl. Math.* **84** 119-44 1991.

[10] Collet, P., Private communication, 1992.

[11] Fineberg, J., Moses, E. & Steinberg, V., Spatially and temporally modulated travelling wave pattern in convecting binary mixtures, *Phys. Rev. Lett.* **61** 838-41 1988.

[12] Heinrich, R., Ahlers, G. & Cannel, D.S., Traveling waves and spatial variation in the convection of a binary mixture, *Phys. Rev. A* **35** 2761-64 1987.

[13] Huerre, P. & Monkewitz, P., Absolute and convective instabilities in free shear layers, *J. Fluid Mech.* **159** 151-68 1985.

[14] Huerre, P. & Monkewitz, P., Local and global instabilities in spatially developing flows, *Annu. Rev. Fluid Mech.* **22** 473-537 1990.

[15] Hunt, R.E. & Crighton, D.G., Instability of flows in spatially developing media, *Proc. R. Soc. Lond. A* **435** 109-129 1991.

[16] Kolodner, P. & Surko, C.M., Weakly non-linear traveling-wave convection, *Phys. Rev. Lett* **61** 842-45 1988.

[17] Kolodner, P., Surko, C.M. & Williams, H., Dynamics of traveling waves near the onset of convection in binary fluid mixtures, *Proc. Conf. Adv. Fluid Turbul.,* Los Alamos, N.M. in press 1989.

[18] Landau, L. & Lifshitz, E.M., *Mechanics of Continuous Media* Moscow : Fizmatgiz (In Russian) 1954.

[19] Landau, L. & Lifshitz, E. M., *Fluid Mechanics* London : Pergamon 1959.

[20] Le Dizès, S., Ph.D thesis in preparation 1992.

[21] Le Dizès, S., Monkewitz P.A. & Huerre. P., Weakly non-linear stability analysis of spatially developing shear flows, *Bull. Am. Phys. Soc.* **36** 2675 1991.

[22] Le Dizès, S, Monkewitz & P. Huerre, *Proceedings of IUTAM Symposium on Bluff-Body Wakes, Dynamics and Instabilities* Göttingen : Springer 1992.

[23] Lifshitz, E.M. & Pitaevskii, L.P., *Physical Kinetics* London : Pergamon 1981.

[24] Manneville, P., *Structures dissipatives, chaos et turbulence* Gif sur Yvette : Aléa-Saclay 1991.

[25] Monkewitz, P.A., Huerre, P. & Chomaz, J.M., Global linear stability analysis of weakly non parallel shear flows, submitted to *J. Fluid Mech.* 1992.

[26] Moses, E., Fineberg, J. & Steinberg, V., Multistability and confined traveling-wave patterns in a convecting binary mixture, *Phys. Rev. A* **35** 2757-60 1987.

[27] Moses, E. & Steinberg, V., Flow patterns and non-linear behavior of travelling waves in a convective binary fluid, *Phys. Rev. A* **34** 693-96 1986.
[28] Soward, A.M.& Jones, C.A., The linear stability of the flow in the narrow gap between two concentric rotating spheres, *Quart. J. Mech. Appl. Math* **36** 19-42 1983.
[29] Sturrock, P.A., Kinematics of growing waves, *Phys. Rev.* **112** 1488-1503 1958.
[30] Sturrock, P. A., Amplifying and evanescent waves, convective and non-convective instabilities, *in Plasma Physics* ed. J.E. Drummond, New-York : McGraw-Hill, 124-42 1961.
[31] Twiss, R. Q., On oscillations in electron streams, *Proc. Phys. Soc. London* Sect. B **64** 654-65 1951*a*.
[32] Twiss, R. Q., On Bailey's theory of amplified circulating polarized waves in an ionized medium, *Phys. Rev.* **84** 448-57 1951*b*.
[33] Twiss, R. Q., Propagation in electronion streams, *Phys. Rev.* **88** 1392-1407 1952.
[34] Winant, C.D. & Browand, F.K., Vortex pairing, the mechanism of turbulent mixing-layer growth at moderate Reynolds number, *J. Fluid Mech.* **63** 237-55 1974.