A numerical analysis of the effects of tree architecture on its dynamics

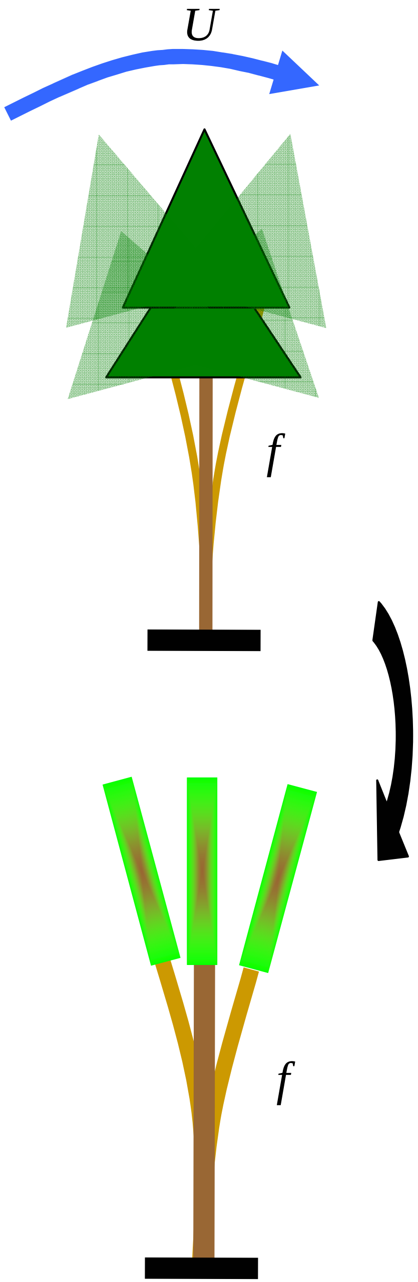
Mathieu Rodriguez[1,2], **Emmanuel de Langre**[1] **& Bruno Moulia**[2]

[1]*Department of Mechanics, LadHyX, Ecole Polytechnique, France*
[2]*Group MECA, INRA, Univ. Blaise Pascal, Clermont-Ferrand, France*

Outline

- Motivations
- Evidence of multimodal dynamics
- Scaling law for tree frequencies
- Prediction of a walnut's natural frequencies
- Conclusion

Motivations



Tree under wind = dynamic interaction.

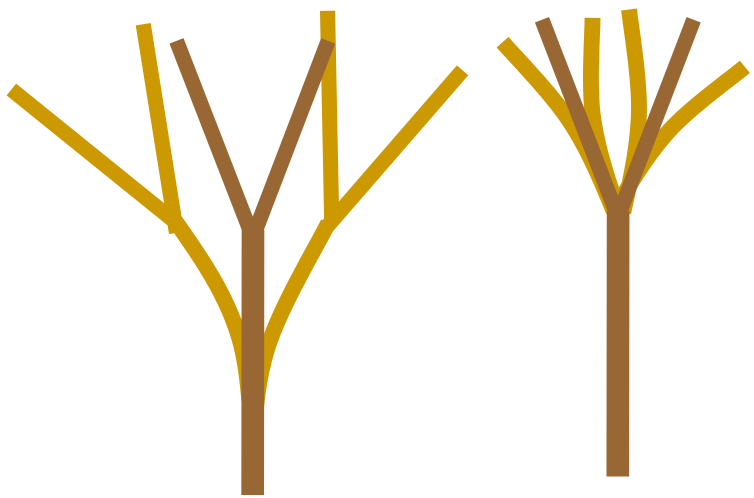
Mayer (1987) – Gardiner & Quine (2000)

Dynamic characteristics of trees :

1D beam model → 1^{st} mode

Gardiner (1991) - Spatz & Zebrowski (2001)

Evidence of multimodal dynamics



Additional frequencies in branches movement.

Moore & Maguire (2005) - Sellier et al. (2006)

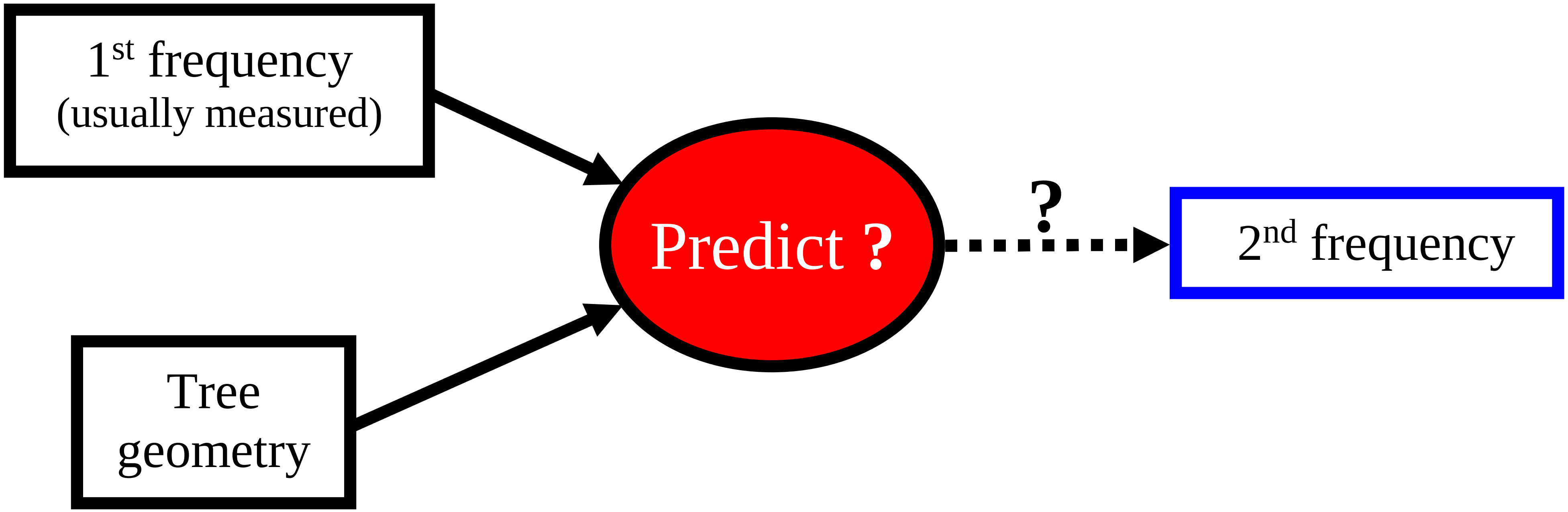
Spatz et al. (2006)

Influence of the branching system :

- Excitation distributed between modes.

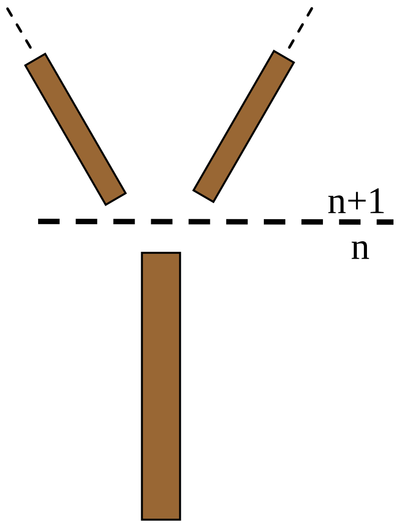
James et al. (2006)

Objective


1st frequency
(usually measured)
Tree
geometry
Predict ?
?
2nd frequency

Geometrical parameters

Branching : $\lambda = \dfrac{Section\,(n+1)}{Section\,(n)}$

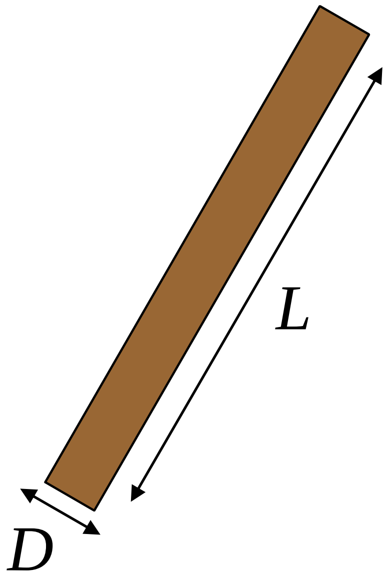


Typical value :

$$\lambda = \frac{1}{number\ of\ branches}$$

Slenderness : $L \approx D^{-1/\beta}$

(Allometric law)



Typical value : β=3/2

(McMahon & Kronauer, 1976

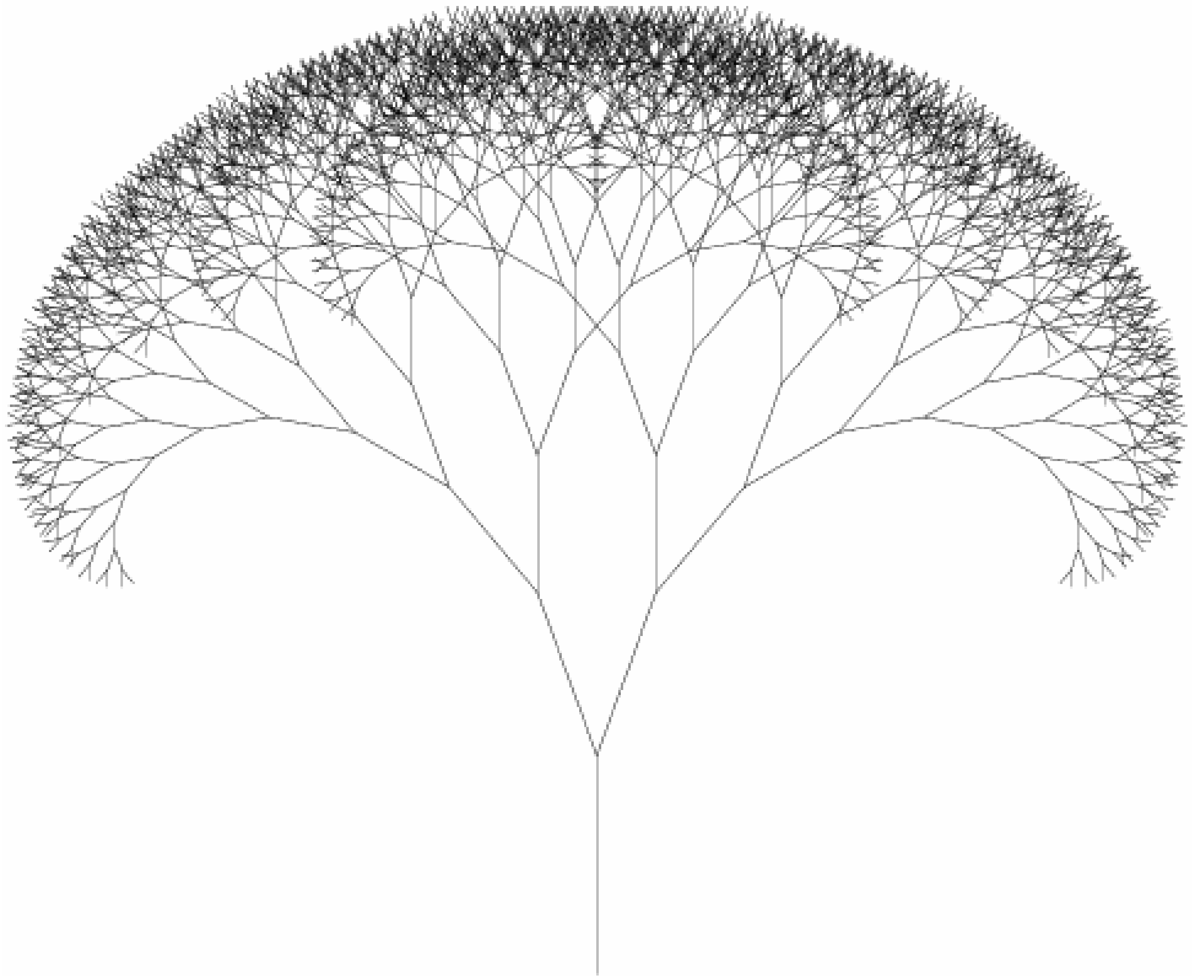
- Moulia & Fournier, 1997)

Example of an idealized branched tree

Generated by auto-similar branching :

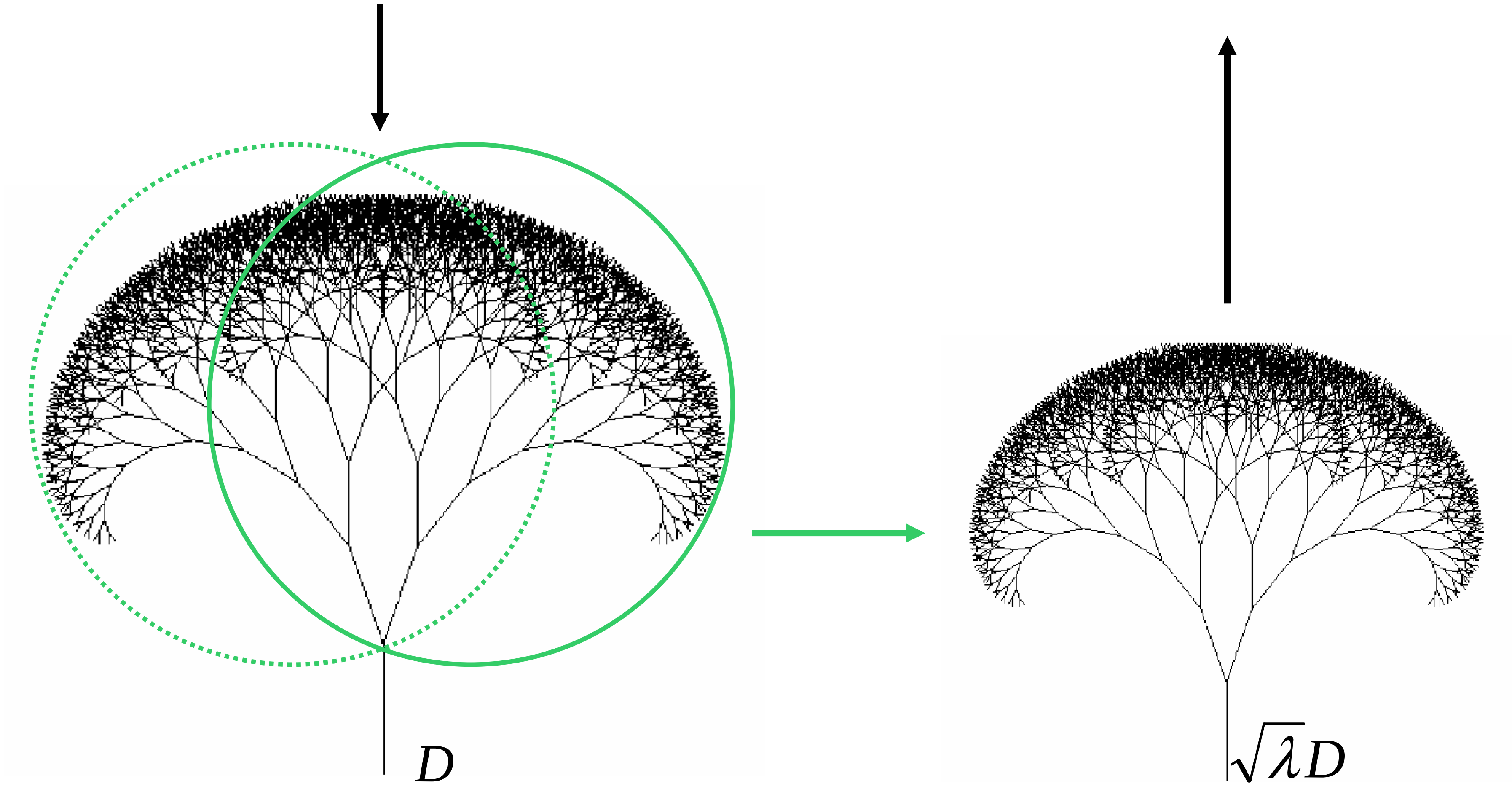
$$\lambda = \frac{1}{2}$$

$$\beta = \frac{3}{2}$$



Determination of modes

1st mode, f_1 ··········?··········► 2nd mode, f_2

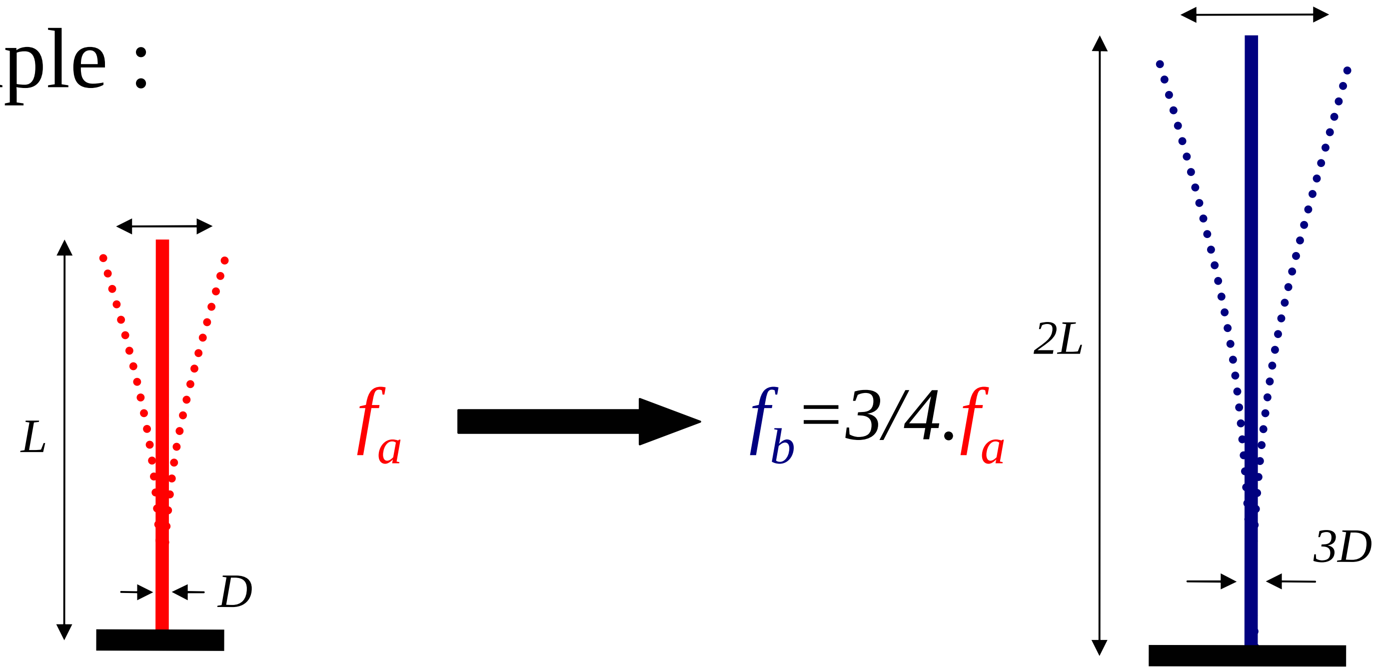


Scaling law for beams frequencies

- Case of a beam system : $f \approx \frac{1}{L^2}\sqrt{\frac{EI}{\rho A}}$ $I \approx D^4$, $A \approx D^2$

Then $$f \approx \frac{D}{L^2}$$

- Example :



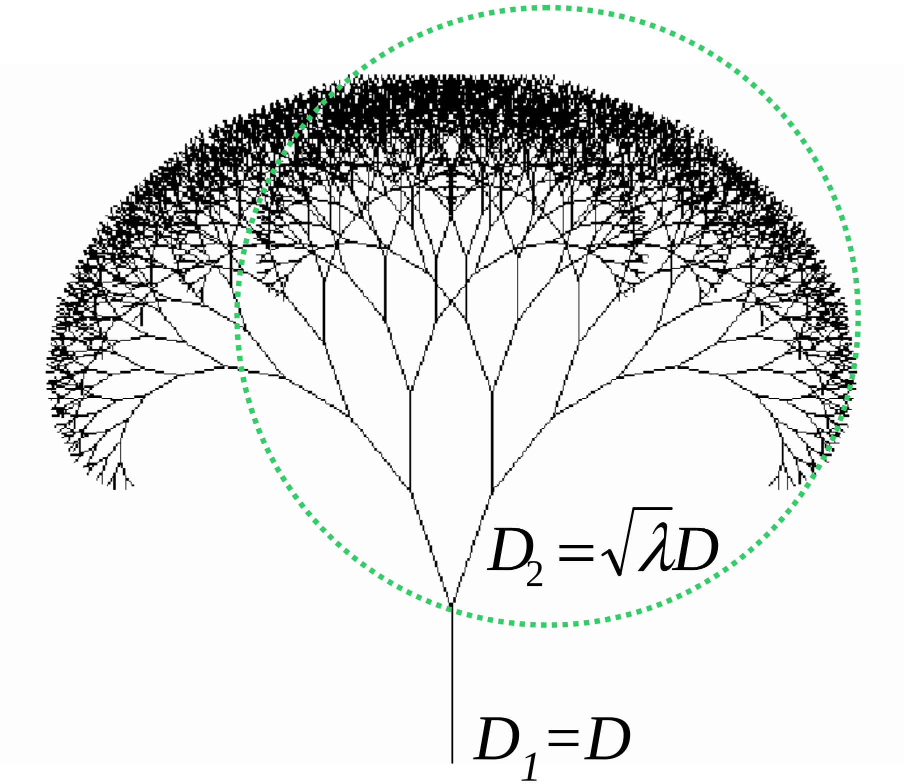
Scaling law for frequency ratio

$$L \approx D^{2/3}$$

$$f \approx \frac{D}{L^2}$$

Then

$$f \approx D^{-1/3}$$



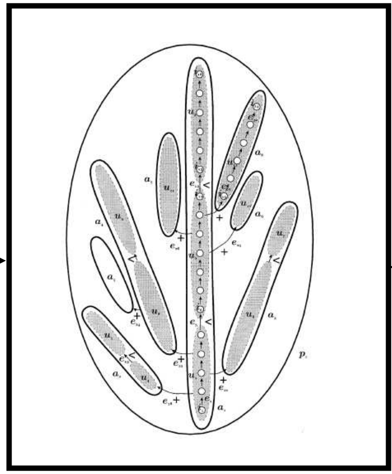
$$\frac{f_2}{f_1} = \left(\sqrt{\lambda}\right)^{-1/3} = 2^{1/6}$$

Analysis of a real tree

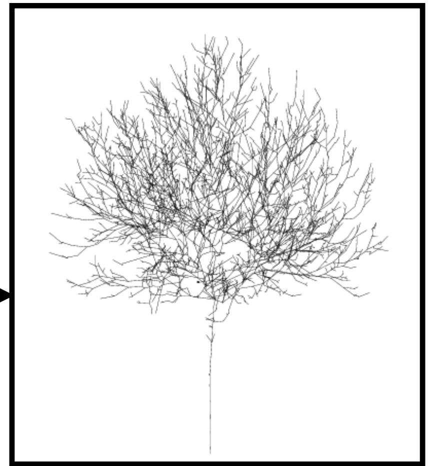
- Walnut digitalized by Godin et al. (1999) :

Methodology

Architecture digitalization



Mesh generation

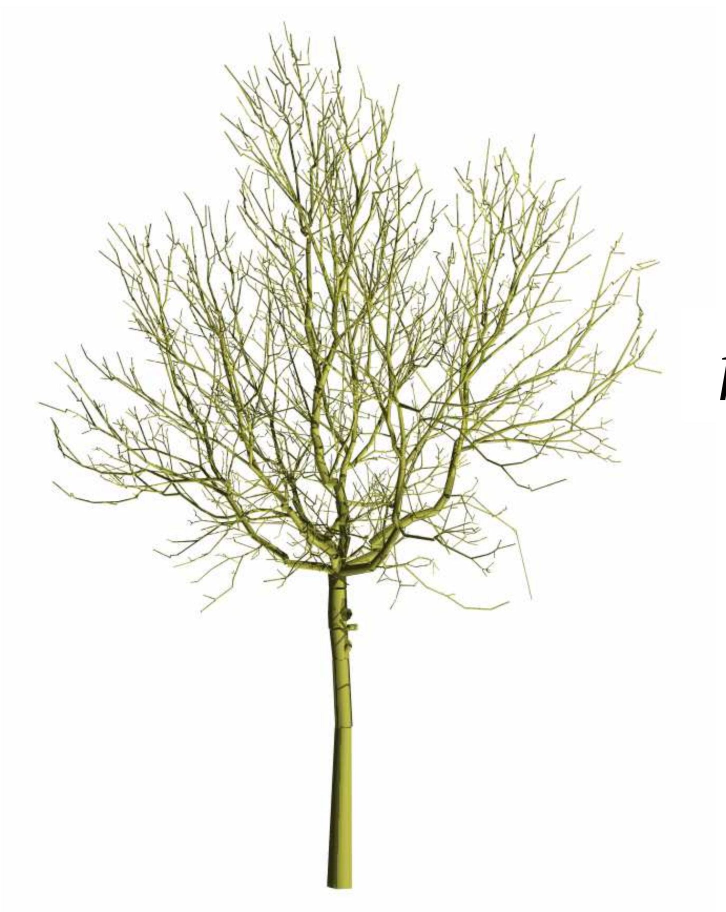


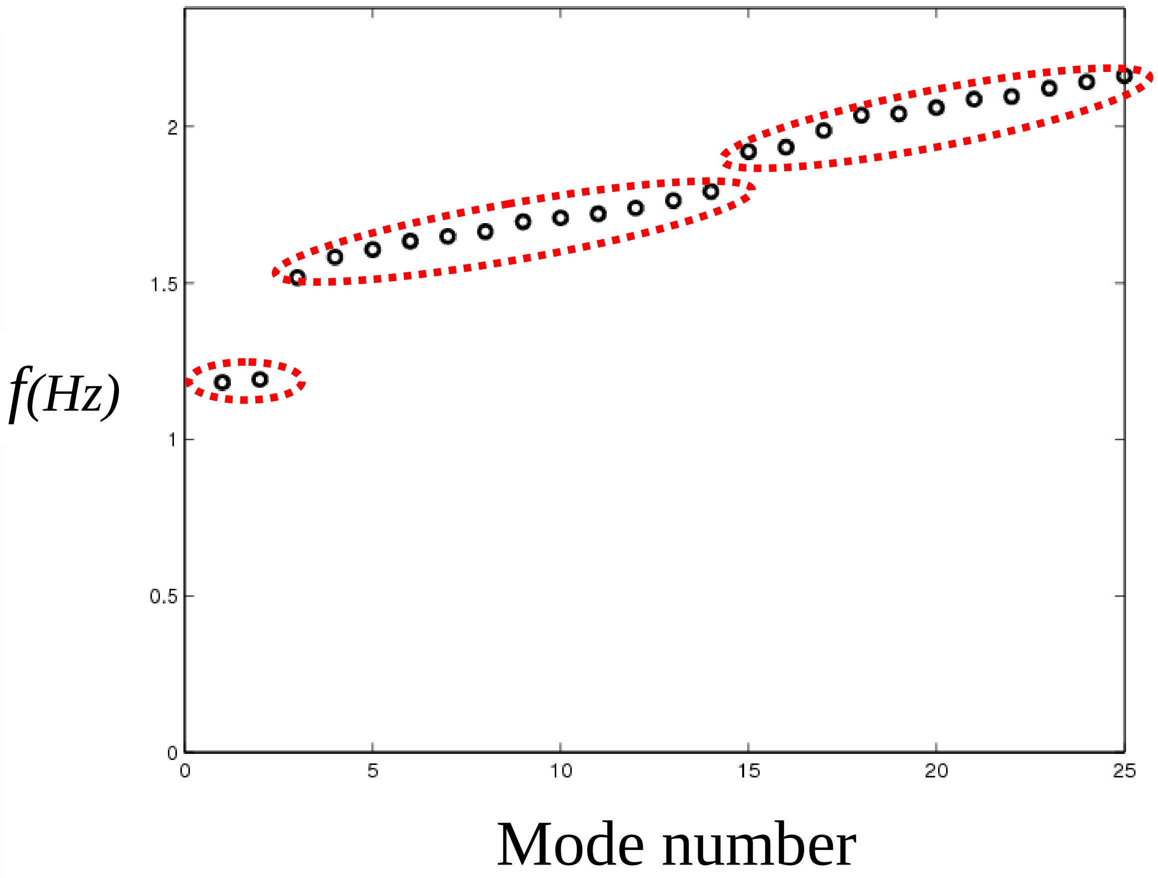
Finite element analysis of the tree dynamics

Modes

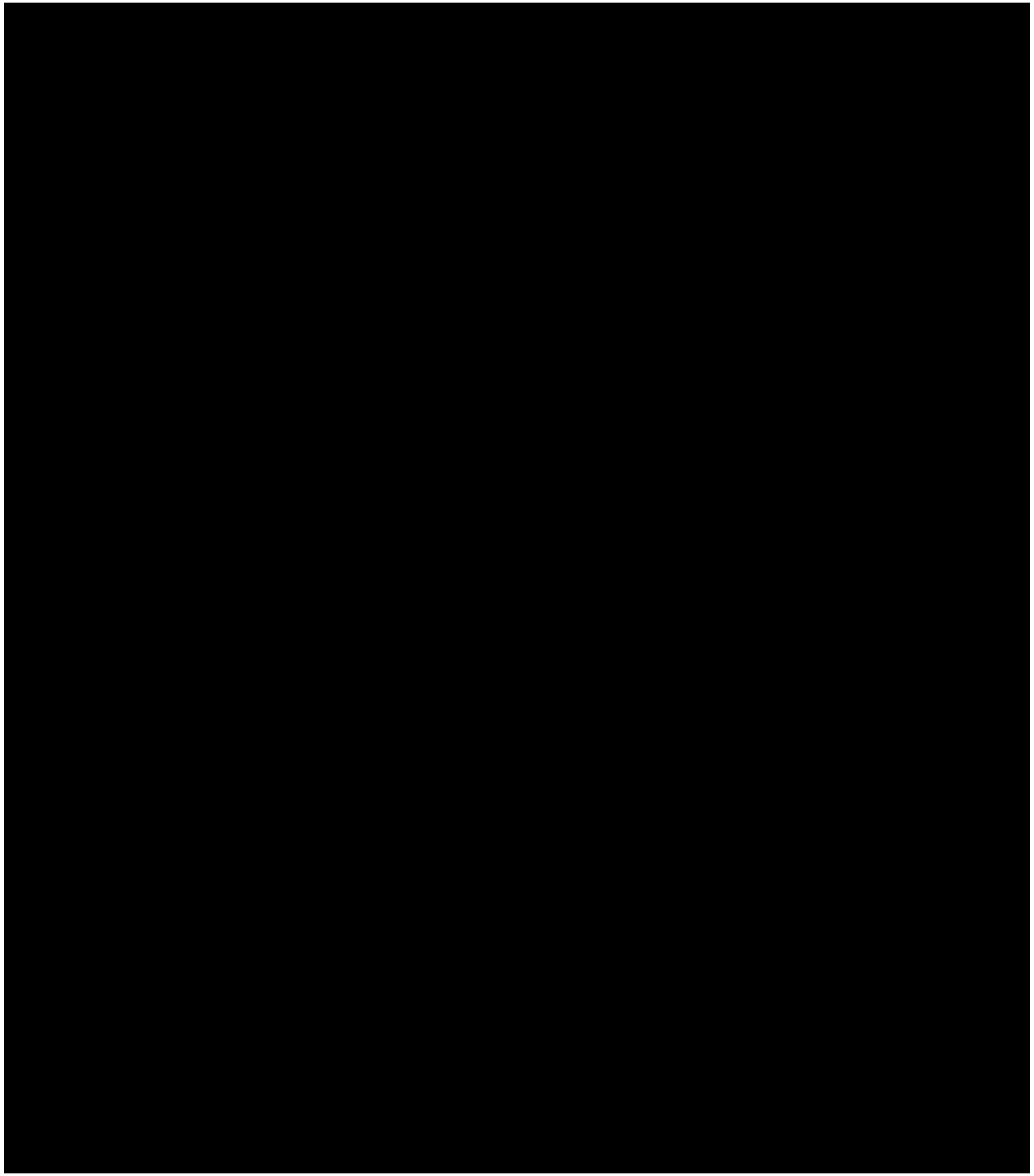
(Shape, frequency, …)

Modes frequencies





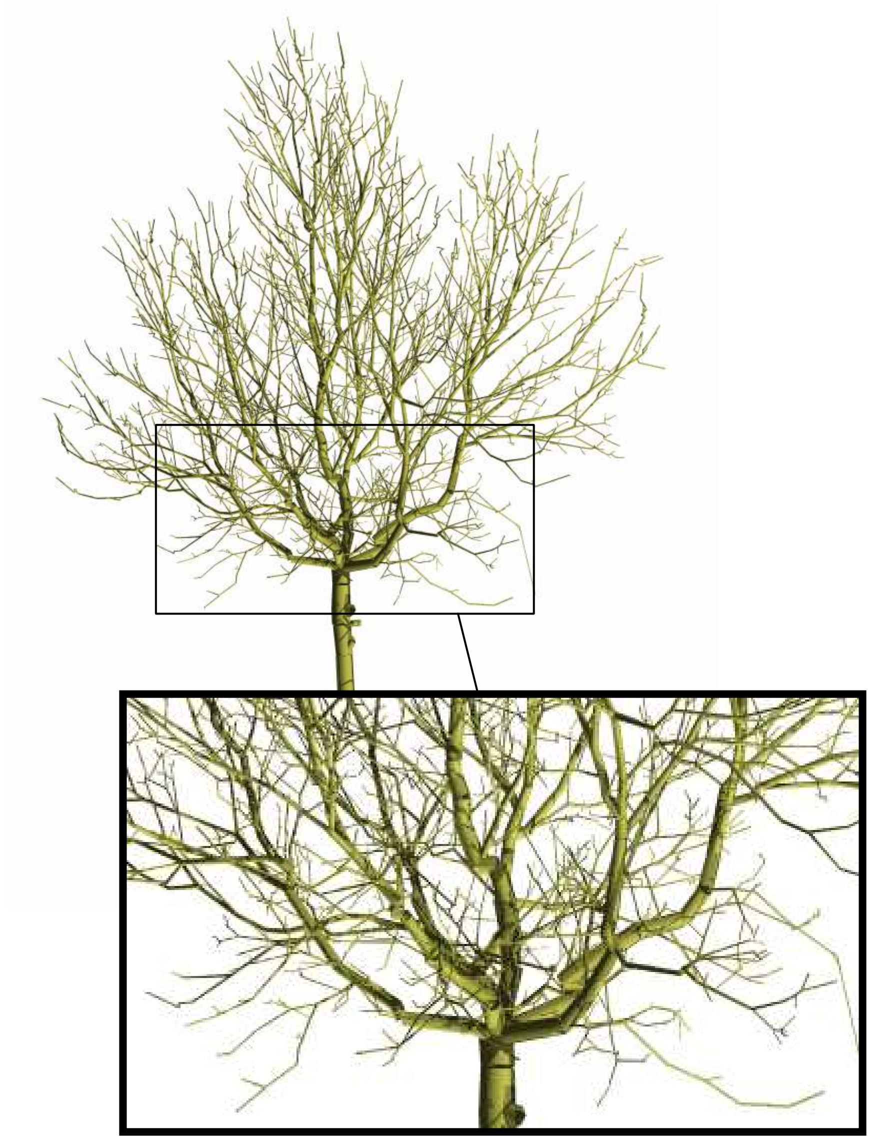
Modes animations

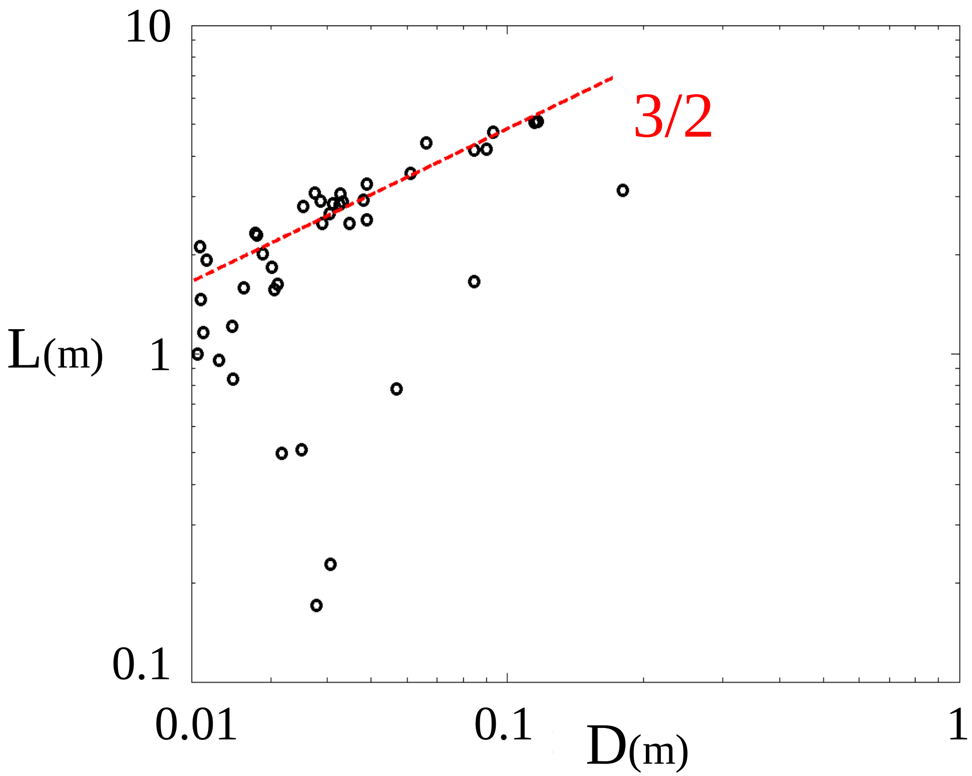


Walnut geometry

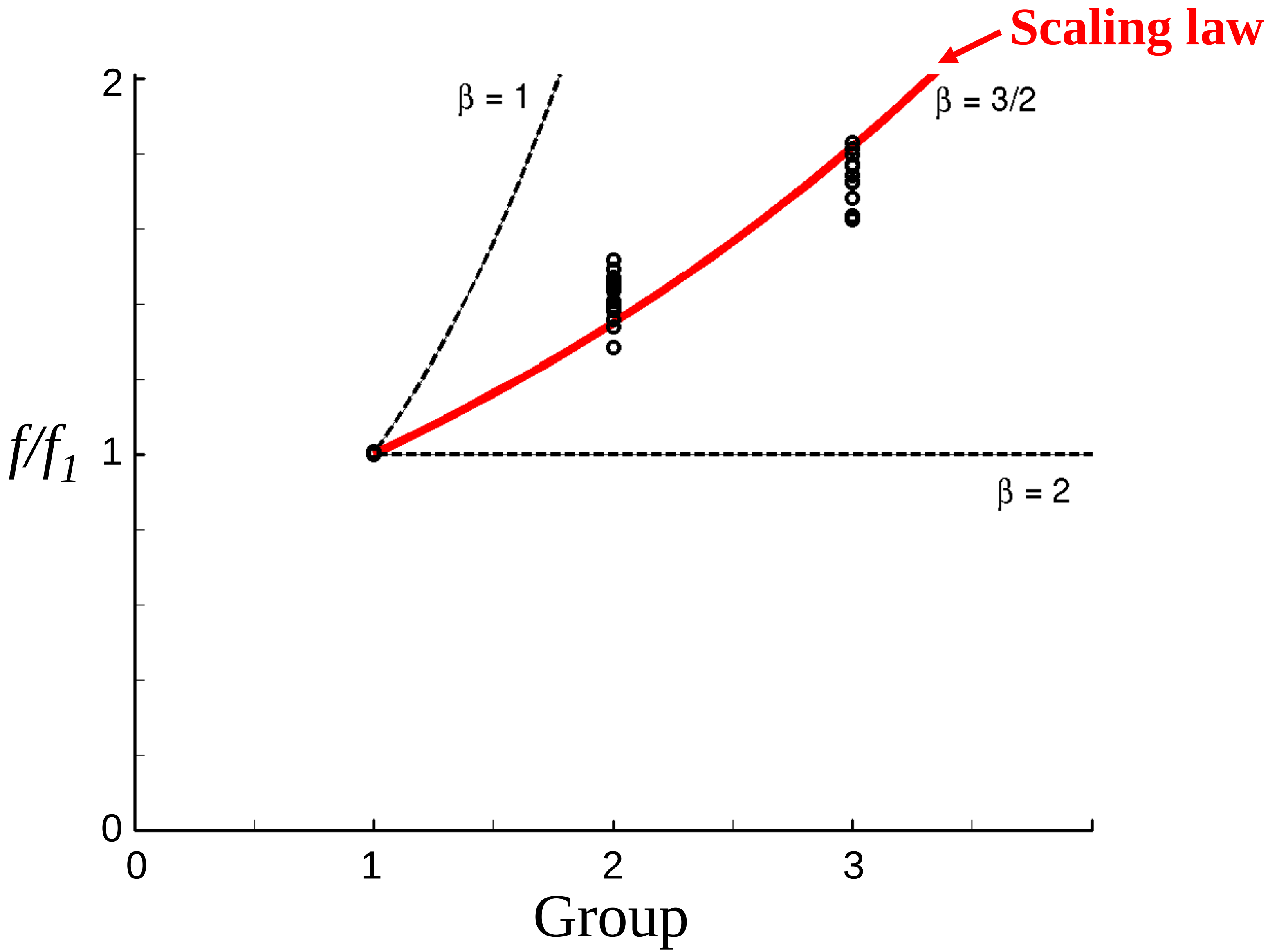
Branching : $\lambda = 1/6$

Slenderness : $\beta = 3/2$

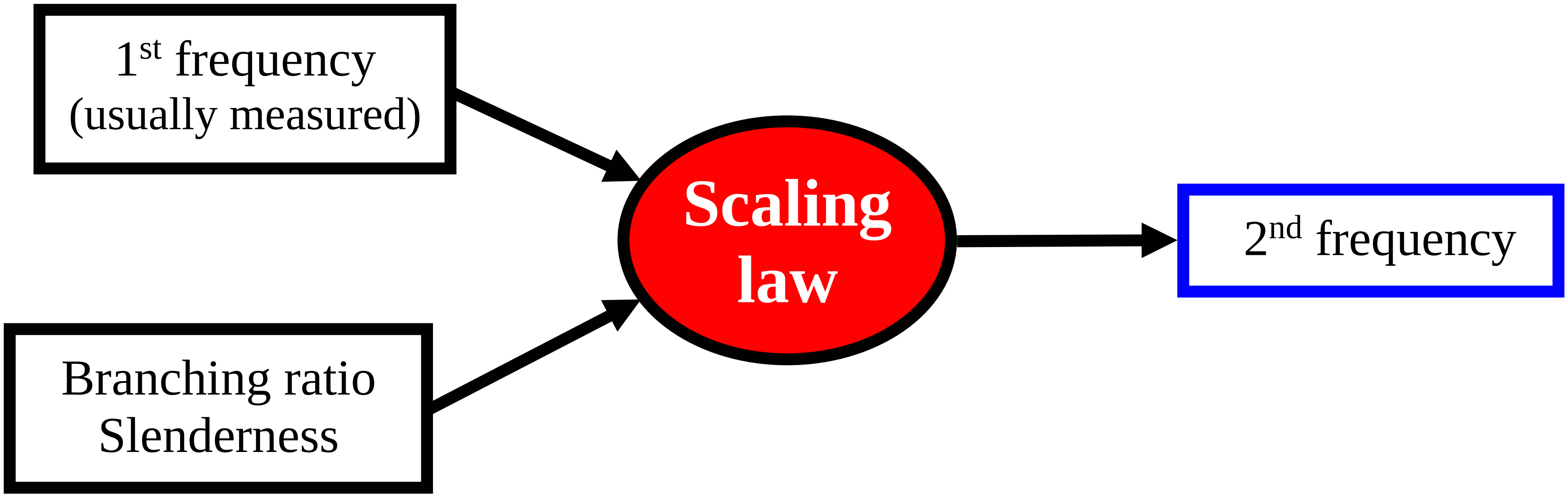




Prediction of frequencies



Conclusion



Other results :

- Prediction of higher modes.
- General scaling law for other tree geometry, eg. 
- Damping.