Proposition de stage 2024/2025

# Rogue waves learning with Physics informed neural networks

Theme: Machine learning and fluid mechanics
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## 1 ML and Physics Context

Physics informed neural network(PINNs) is a new paradigm based on neural networks, which has been proposed a few years ago[1] for solving PDE. Typically, $D$ being some partial differential operator and $f$ some source term, the solution $u$ of the PDE

$$Du = f$$

on some domain $\Omega \subset \mathbb{R}^d$ is sought in the form of a neural network $u_\theta$, taking as input any $x \in \Omega$ and yielding as output the value $u_\theta(x)$ of the solution; $\theta \in \mathbb{R}^P$ is the vector of parameters used to define the NN. The PDE can be explicitly encoded into the loss of the NN

$$\mathcal{L}(\theta) = \|Du_\theta - f\|^2$$

which is minimized in order to find the solution, because all derivatives of $u_\theta(x)$ w.r.t. $x$ are exactly accessible thanks to auto-differentiation which is a key component in modern machine learning frameworks used to train NN. This framework is very appealing as it is mesh-free and very flexible: arbitrary parameterization of $u_\theta$ are allowed, the choice of training points replacing the mesh is free, and it gives the possibility of performing data assimilation with observations. Still, using this methods for non-smooth problems (when solutions displays shocks or point singularities for instance) constitutes a challenge, because the corresponding optimization problem is badly conditioned due to the multi-scale nature of the problem. A way to deal with such difficulties as been proposed in [3] in the form of en efficient natural gradient method, corresponding basically to use the green function of the PDE restricted to the empirical tangent space of the NN manifold.

An example of a challenging problem involving singularities is the case of conserved flow exhibiting finite-time localized singularities, which manifest in real-world systems as rogue waves. A simplified setting for such systems displaying rogue waves is provided by the self-focusing non-linear Schrödinger equation (NLS).

## 2 Objectives

The PDE to be considered in this internship is the self-focusing NLS:

$$i\frac{\partial\psi}{\partial t} = -\frac{\alpha}{2}\nabla^2\psi - g|\psi|^n - i\nu\Delta^2\psi + f_{k_0}(\mathbf{x}, t) \tag{1}$$

where $\psi$ is a complex field in spatial dimension $D$. The parameter $\alpha$ quantifies the dispersion and $g$ the non-linear strength while $\nu$ represent the viscosity responsible of dissipation The goal of the internship is twofold:

- to train a PINNs for solving (1). Owing to the flexibility of PINNS we will exploit the fact that the shape of the singularities is known analytically, so that the functional space of the PINN can be enriched some appropriate dictionary of functions.
- Study the different regimes depending on the balance between non-linearity and viscosity.

## 3 Practical conditions of the Internship

The stage will take place in the context of a collaboration between Ladhyx and LISN, the intern will be indifferently located at LISN or at Ladhyx.

### Further reading


[1 ] Raissi, Perdikaris and Karniadakis, “Physics-informed neural networks: a deep learning framework for solving forward and inverse problems involving PDE”, journal of computational physics (2019)

[2 ] Josserand, Pomeau and Rica, “Finite-time localized singularities as a mechanism for turbulent dissipation” Phys.Rev.Fluids (2020)

[3 ] Schwencke,Furtlehner "ANaGRAM: A Natural Gradient Method for efficient PINNs learning", submitted (2024)